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$$
\begin{aligned}
&s+a=y^2+2yz+z^2+x(2x+y+z)=(y+z)^2+x^2=2x^2\,;\\
\text{similarly, }&s+b= \qquad\qquad\qquad\qquad\qquad\qquad\qquad =2y^2\,;\\
&s+c= \qquad\qquad\qquad\qquad\qquad\qquad\qquad =2z^2.
\end{aligned}
$$

Hence $\sqrt{(s+a)}+\sqrt{(s+b)}+\sqrt{(s+c)}=(x+y+z)\sqrt{2}=0$, an equation involving only a, b, c.

If we wish to express this relation without radicals we transpose, square, and reduce; whence $a-s=\sqrt{[(s+b)(s+c)]}$, whence

$$
\begin{aligned}
s^2+as&=a^2-bc\\
s^2+bs&=b^2-ca\\
s^2+cs&=c^2-ab.
\end{aligned}
$$

Whence $4s^2=a^2+b^2+c^2-(ab+bc+ca)$; and therefore

$$a^2+b^2+c^2+3(ab+bc+ca)=0.$$

This may also be written

$$\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}+3\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=0.$$

Example. Let $x=+1$, $y=+2$, $z=-3$; then $a=-5$, $b=+1$, $c=+11$.

Excellent solutions of this problem were received from *F. L. SAWYER*, *LON C. WALKER*, *JOSIAH H. DRUMMOND*, *J. K. ELLWOOD*, *J. SCHEFFER*, and *G. B. M. ZERR*.

Mr. Baker sent in neat solutions of problems 151 and 152.

154. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Show that the equation, $x^4+qx^2+s=0\ldots.(1)$, can not have three *equal* roots.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio University, Athens, Ohio.

By the usual theory, the conditions that $ax^4+4bx^3+6cx^2+4dx+e=0..(1)$ shall have three equal roots are

$$
\begin{aligned}
&ae-4bd+3c^2=0\ldots.(2),\\
&ad^2+eb^2+c^3-ace-2bcd=0\ldots.(3).
\end{aligned}
$$

In the given equation, $x^4+qx^2+s=0\ldots.(4)$, $a=1$, $b=0$, $c=\frac{1}{6}q$, $d=0$, $e=s$, and (2) and (3) become

$$s+\frac{q^2}{12}=0\ldots.(5),\quad \frac{q^3}{36}-qs=0\ldots.(6).$$

Eliminating s from (5) and (6) gives $q=0$, which is inconsistent with the supposition.

Also solved similarly by *G. B. M. ZERR*, *F. L. SAWYER*, *JOSIAH H. DRUMMOMD*, *J. R. HITT*, and *HARRY S. VANDIVER*.

155. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

If the roots of the cubic $x^2+3px^2+3qx+r=0$ be in harmonical progression, $2q^3=r(3pq-r)$.

Solution by HOMER R. HIGLEY, M. S., State Normal School, East Stroudsburg, Pa.; J. R. HITT, Coronal Institute, San Marcos, Tex.; H. S. VANDIVER, Bala, Pa.; and the PROPOSER.

If α, β, γ be the roots of a cubic, we have

$$x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\alpha\gamma+\beta\gamma)x-\alpha\beta\gamma=0 \ldots .(1).$$

By the condition of the problem,

$$\beta=\frac{2\alpha\gamma}{\alpha+\gamma}, \text{ or } \alpha\beta+\alpha\gamma+\beta\gamma=3\alpha\gamma \ldots .(2),$$

and (1) becomes

$$x^3-(\alpha+\beta+\gamma)x^2+3\alpha\gamma x-\alpha\beta\gamma=0 \ldots .(3).$$

Comparing (3) and

$$x^3+3px^2+3qx+r=0 \ldots .(4),$$

$$\alpha+\beta+\gamma=-3p \ldots .(5), \quad \alpha\gamma=q \ldots .(6), \quad \text{and } \alpha\beta\gamma=-r \ldots .(7).$$

From (6) and (7), $\beta=-r/q \ldots .(8)$.
From (5) and (8), $\alpha+\gamma=r/q-3p \ldots .(9)$.
Eliminating α, β, γ from (2) by use of (6), (8), (9), $2q^3=r(3pq-r)$.

Also solved by *J. SCHEFFER*, *LON C. WALKER*, *JOSIAH H. DRUMMOND*, *F. L. SAWYER*, and *G. B. M. ZERR*.

GEOMETRY

183. Proposed by S. F. NORRIS, Professor of Mathematics and Astronomy in Baltimore City College, Baltimore, Md.

Two quadrilaterals having three sides of the one equal to the three corresponding sides of the other, each to each, and the two corresponding angles adjacent to the unknown sides equal, each to each, are equal figures. [*Olney's Geometry*, page 129.]

This is problem 169, an erroneous demonstration of which was published in the April number of the current volume of the MONTHLY. The fallacy in the demonstration is pointed out in Dr. Halsted's article, "Proving the False," on page 129. ED.